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ABSTRACT

This paper presents a finite element solution algorithm for three-dimensional free surface flows for die casting applications. The problems of interest present unusual challenges for both the physical modelling and the solution algorithm. High Reynolds number transient flows with free surfaces have to be computed on complex three-dimensional geometries. In this work, a segregated algorithm is used to solve the Navier-Stokes, turbulence and front tracking equations. The Streamline-upwind/Petrov-Galerkin method is used to obtain stable solutions to convection dominated problems. Turbulence is modelled using either a one-equation turbulence model or the $k - \epsilon$ two-equation model with wall functions. The position of the flow front in the mold cavity is computed using a level set approach. Finally, equations are integrated in time using an implicit Euler scheme.

In this work special attention is given to the solution of the free surface flow in the presence of gravity. When using a fixed mesh, the fluid/air interface is generally located inside mesh elements. A simple interpolation leads to inappropriate treatment of gravity forces, pressure gradient and inertia within partially filled elements and generates spurious oscillations which are caused by discontinuities of the gravity forces and pressure gradient across the interface. In this work we use enriched shape functions for the pressure that introduce a discontinuity in the normal pressure gradient at the interface [1]. The additional pressure degree of freedom is eliminated at the elementary system level by static condensation. Therefore the finite element degrees of freedom and the size of the resulting matrix system remain the same, making implementation much easier than for extended finite element methods.

The enriched pressure discretization is first validated on a series of simple free surface flows and then applied to a die casting problem. The first test problem is the collapse of a column of liquid for which experimental measurements are available [2]. Figure 1 shows the solution for different values of the dimensionless time. The free surface is well captured and mesh refinement studies indicate that the solution depends very little on the mesh element size. Figure 2 compares the horizontal displacement of the liquid as given by a laminar, one-equation model and $k - \epsilon$ model with the experimental measurements. The agreement is very good and a slight dependence on the turbulence model is observed.

The method is then applied to the solution of a gravity casting application [3]. The filling pattern when solving with the one-equation model is shown in Figure 3. As can be seen the solution approach deals extremely well with very deformed free surfaces. It avoids oscillations in the velocity field whitout the need of remeshing. It is able to capture jets, folding of the free surface and sharp changes in the free surface. The methodology presents the robustness and cost effectiveness needed to tackle complex industrial applications.

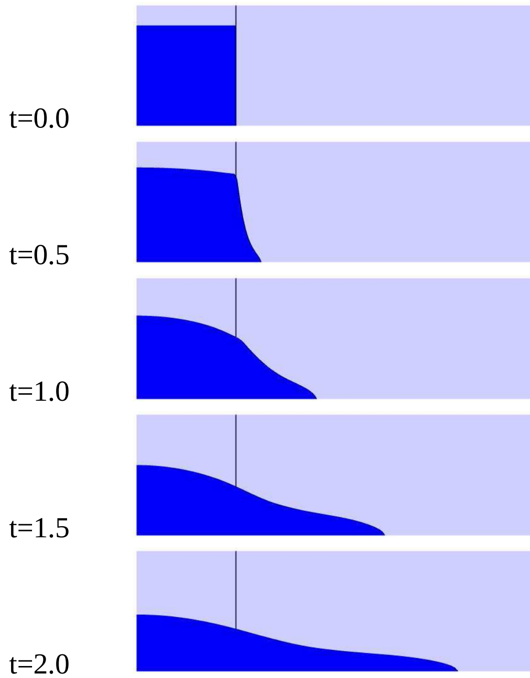

Figure 1. Collapse of a column of liquid

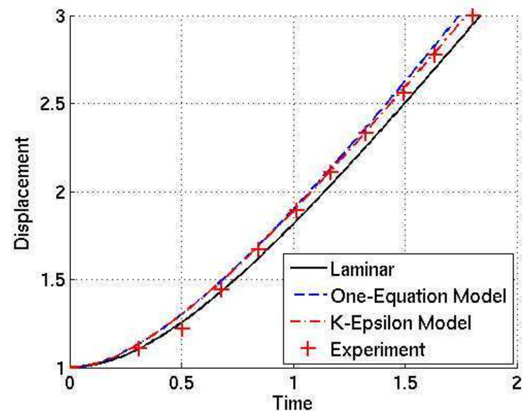

Figure 2. Horizontal displacement

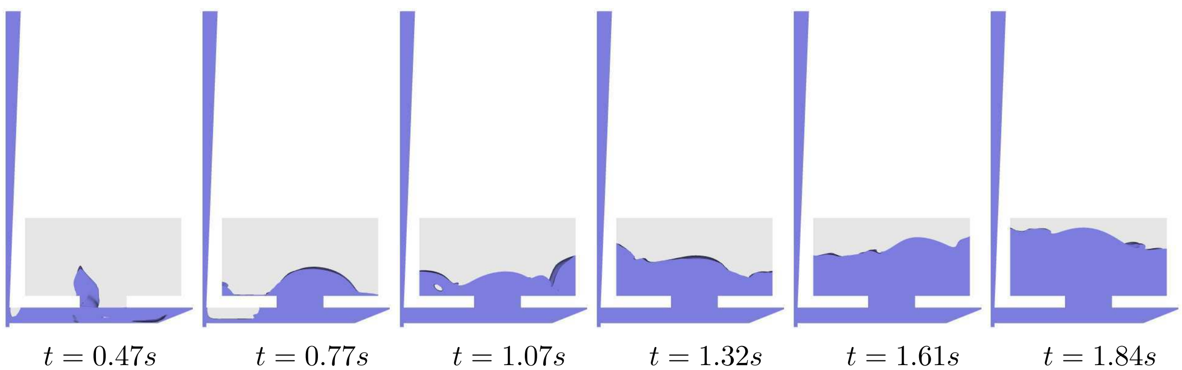

Figure 3. Gravity die casting application

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